Repeated Four Phase Projection on a Single Sheet

A Lean Checked Finite Model and Its Open Prime Frequency Bridge

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Abstract

We isolate a finite geometric model in which four quarter phases are repeatedly projected onto one common half-width sheet. In cycle q, the physical phase is 2πq + kπ/2 and the projection subtracts 2πq before dividing by 4π. The four projected readings are 1/8, 1/4, 3/8 and 1/2, independently of q. The fourth quarter reaches 2π(q + 1) and projects to 1/2. The four complex phase layers cancel at any common complex amplitude.

The Lean source proves finite algebraic identities and a conditional localization theorem. The real localization theorem assumes both observer closure and identification of the horizontal coordinate with the sheet projection. No concrete prime-frequency observer or proof of these hypotheses for actual nontrivial zeta zeros is supplied. These analytic obligations remain OPEN, and this work does not prove the Riemann hypothesis. A finite prime-phase table is COMPUTED separately from Lean.

Keywords: Lean 4, formal verification, roots of unity, phase projection, prime frequencies, Riemann zeta function.

1 Introduction

For s = beta + i gamma, each prime contribution separates radial decay from logarithmic phase:

$$p^{-s} = p^{-\beta}\exp(-i\gamma \ \log \ p)$$

This construction projects four quarter turns onto a fixed half-width sheet. Its endpoint is a consequence of the chosen normalization. The contribution is an explicit finite formal model and a precisely stated conditional interface. A zeta application would require an independently defined observer, prime-frequency provenance and a proof of the entire displayed hypothesis at every nontrivial zero. These requirements are not established here.

- Exact single sheet projection for every cycle number.
- A four periodic reading sequence with closure at 1/2.
- Physical closure at 2π(q + 1).
- Four-phase cancellation for every common complex amplitude.
- A centered defect and conditional real localization theorem.
- A falsifiability check separating raw prime phases from an aggregate observer.

2 Prime phases and model scope

For real part of s greater than 1, the Euler product is

$$\zeta(s) = \prod_{p \ prime}(1 - p^{-s})^{-1}$$

Write θ_p(γ) = γ log p modulo 2π, with period T_p = 2π/log p. This is the positive phase convention used for the table; the factor p^(−iγ) has phase −θ_p. The quarter grid is invariant under this sign reversal. For distinct primes p and r, log p/log r is irrational: a rational ratio would imply p^b = r^a for positive integers a and b, contrary to unique factorization. Hence their periods are incommensurate.

The finite four-phase states are a proposed model for an aggregate observer. They are not identified with the raw phases of individual primes. The absolutely convergent Euler product is available for Re(s) > 1, not throughout the critical strip. Any proposed prime-frequency observer there must specify and justify its continuation or regularization. No such construction is included.

3 Single sheet four phase projection

$$\varphi_{q,k} = 2\pi q + \frac{k\pi}{2}$$

$$\Pi_q(\theta) = \frac{\theta - 2\pi q}{4\pi}$$

Direct substitution proves the projected quarter law

$$\Pi_q(\varphi_{q,k}) = \frac{k}{8}$$

The reading is independent of q. The four cumulative readings below are internal stages of one projected orbit, not locations of four distinct zeta zeros. In units of π, the normalized phase is x(q,k) = 2q + k/2 in $\mathbb{Q}$ and the normalized projection is P_q(x) = (x − 2q)/4. Direct substitution gives P_q(x(q,k)) = k/8.

Quarter k	Physical phase	Projected reading
1	2πq + π/2	1/8
2	2πq + π	1/4
3	2πq + 3π/2	3/8
4	2π(q + 1)	1/2

$$a_n = \frac{(n \bmod 4) + 1}{8}, a_{n+4} = a_n$$

The sequence repeats after four steps and has no ordinary limit: the subsequences a_(4m) and a_(4m+3) are constantly 1/8 and 1/2. One half is the endpoint of each completed cycle, not a limiting value of the whole sequence. The factor 4π in the projection definition fixes this endpoint; it is not derived from properties of zeta zeros.

4 Overlapping complex layers

$$1 + i - 1 - i = 0$$

More generally, for every a ∈ $\mathbb{C}$, a·1 + a·i + a·(−1) + a·(−i) = 0, by factoring out a. This is the additional theorem four_phase_cancel_smul in source version 1.1.0. A common amplitude is sufficient for cancellation; it is not necessary. Independently weighted layers A0 + iA1 − A2 − iA3 require suitable relations, for example A0 = A2 and A1 = A3. The observer hypothesis below contains no amplitude variables and proves no such relations for a prime-frequency realization.

5 Transverse defect and conditional localization

$$D(\beta) = \left(\beta - \frac{1}{2}\right)^2$$

In the Lean source, this defect is defined for β ∈ $\mathbb{Q}$. It is nonnegative and vanishes exactly at β = 1/2. It is a defined squared displacement, not a zeta-derived energy. The real-valued localization theorem is separate.

Let Z(β,γ) be any predicate on $\mathbb{R}^2$ and Θ(β,γ) any real-valued observer. Assume that for every β,γ ∈ ℝ satisfying Z(β,γ), there exists q ∈ ℕ for which both equalities hold:

$$\Theta(\beta,\gamma) = 2\pi(q + 1), \qquad \beta = \Pi_q(\Theta(\beta,\gamma))$$

Then every instance of Z satisfies β = 1/2. Proof: substitute Θ(β,γ) = φ(q,4) into β = Π_q(Θ(β,γ)); the real identity Π_q(φ(q,4)) = 1/2 finishes the argument. Both equalities are required. The theorem real_localization_of_two_pi_closure, including its hypothesis and proof, is unchanged from published version 1.0.0.

Logical strength of the abstract interface (informal analysis). With the observer unconstrained, existence of an observer satisfying the complete hypothesis is equivalent to localization of Z at β = 1/2. One direction is the formal theorem. Conversely, if Z localizes at 1/2, choose Θ = 2π and q = 0. A useful analytic realization therefore needs independently justified structure; the abstract existential formulation alone is not a weaker route to localization.

6 Lean formalization

The unchanged source version 1.1.0 has 19 theorems and 7 definitions. The original audit lists 13 principal declarations. The rational interface localization_of_all_closed uses β ∈ ℚ; the real interface below uses β ∈ ℝ. Only the latter directly accommodates an arbitrary real coordinate. The following table summarizes the principal declarations.

Declaration	Formal content
projected_quarter	Rational projection k/8
same_sheet	Cycle independence
physical_projected_quarter	Real projection k/8
physical_cycle_closure	Fourth quarter 2π(q + 1)
physical_closed_projection	Real closure projects to 1/2
orbitReading_zero / one / two / three	Four explicit rational readings
orbitReading_add_four	Period four
closedReading_eq_half	Rational closure at 1/2
orbitReading_three_is_closure	Fourth reading equals closure
four_phase_cancel	Unit-amplitude cancellation
four_phase_cancel_smul	Any common complex amplitude
transverseDefect_nonneg	Nonnegative rational defect
transverseDefect_eq_zero_iff	Rational defect vanishes iff β = 1/2
beta_eq_half_of_closed_projection	Pointwise rational consequence
localization_of_all_closed	Conditional rational interface
real_localization_of_two_pi_closure	Conditional real observer interface

The dated verification receipt records Lean 4.19.0, Mathlib v4.19.0, successful builds and source/audit checks with --trust=0. The source contains no sorry, admit or user-declared axiom. The separate current validation report records the checks performed for this revision and their exact scope. Kernel checking certifies the encoded implications, not that their analytic hypotheses hold for zeta zeros.

7 Numerical falsifiability check

At the first known zero ordinate $\gamma_1 \approx 14.134725141734693790 46$, compute u_p = frac($\gamma_1$ log p/(2π)). The five displayed values are unchanged from versions 1.0.0 and 1.1.0 and have been recomputed with mpmath at 80 decimal digits. Each agrees with rounding to ten digits after the decimal point:

Prime p	Computed u p
2	0.5593117823
3	0.4714507019
5	0.6206098361
13	0.7701392523
23	0.0536434561

These are COMPUTED floating-point values, not Lean theorems or interval-certified bounds. The smallest computed distance to the quarter grid among these examples is approximately 0.02013925226 (p = 13). They provide a finite diagnostic against identifying these individual prime phases with quarter points. They neither construct an aggregate observer nor prove a statement about all zeros. The calculation script and detailed results accompany this revision.

8 Open analytic requirements

A zeta specialization requires: (1) an observer defined in the critical strip with justified analytic continuation or regularization; (2) proved prime-frequency provenance; (3) a proof that every nontrivial zero yields Θ(β,γ) = 2π(q + 1) for some $q \in \mathbb{N}$; and (4) a proof of the same-witness identification β = Π_q(Θ(β,γ)). The complete hypothesis must be established without assuming β = 1/2 or an equivalent formulation. Neither the observer nor these proofs is supplied.

$$\zeta(\beta + i\gamma) = 0 \ \wedge \ 0 < \beta < 1$$

$$\Rightarrow \exists q \in \mathbb{N}, \ \Theta(\beta,\gamma) = 2\pi(q + 1) \ \wedge \ \beta = \Pi_q(\Theta(\beta,\gamma))$$

The displayed implication remains OPEN. Renaming it an analytic input does not establish it. Closure alone is insufficient: the constant observer Θ = 2π closes with q = 0 even when the abstract predicate selects β = 3/4; in that case the required projection identity fails. A successful application must prove both closure and coordinate identification for the actual zeta-zero predicate.

9 Claim status and trust boundary

Result	Status	Boundary
Projection and closure	PROVED	Defined rational and real models
Period four	PROVED	Finite reading sequence
Common-amplitude cancellation	PROVED	One common $a \in \mathbb{C}$
Centered defect	PROVED	$\beta \in \mathbb{Q}$
Localization implication	PROVED	Conditional on the full hypothesis
Prime-phase table	COMPUTED	Finite floating-point calculation
Prime-frequency observer	OPEN	Construction and provenance missing

Result	Status	Boundary
Zero to closure and identification	OPEN	Both equalities must be proved
Zeta localization application	CONDITIONAL	Requires the open analytic inputs
Riemann hypothesis	OPEN	Not proved by this work

PROVED refers to the finite statements and to the conditional implication as a theorem. It does not mean that its antecedent has been proved for zeta zeros. COMPUTED refers only to the numerical table. OPEN identifies the missing observer construction, prime-frequency provenance and full zero-to-closure-and-identification law. The analytic discussion and the explanatory examples are not additional Lean declarations.

10 Reproducibility and conclusion

This edition is published by amnai.org as AMNAI Research Report 001, version 1.2.0. It includes the unchanged Lean source and dependency pins from code package 1.1.0, a public verification summary, reproducible numerical calculations, citation metadata and checksums. The DOCX is the editable manuscript source and the PDF is rendered from it. The earlier version at DOI 10.5281/zenodo.23687973 was withdrawn on 1 November 2026. This corrected edition preserves the mathematical statements and distinguishes the report edition from the unchanged formal source version. The publication page records the current DOI registration status.

The finite identities and the conditional localization implication are exact statements about the defined model. The only added formal theorem since published version 1.0.0 is cancellation at a common complex amplitude. No new result locating actual zeta zeros is established. Construction and justification of the prime-frequency observer and its full closure-and-identification law remain open; this work does not prove the Riemann hypothesis.

References

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